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TURBULENCE STUDIES IN CONJUNCTION WITH THE DETERMINATION OF THE ACOUSTIC CHARACTERISTICS OF A JET

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ABSTRACT

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Experimental determination of the components of flow fluctuation rate, the turbulent vortex volume, and the turbulent fluctuation frequency in order to make use of the Lighthill equation in determining the acoustic power of a free turbulent jet. The equation is used to obtain from jet velocity fluctuation components the spectra of the longitudinal, tangential and radial components and acoustic power of the jet.

Author

Fundamental equations describing the acoustic field generated by an isothermal turbulent jet of incompressible fluid were derived by Lighthill (refs. 1 and 2). Using the equation of motion for the fluid and the equation of discontinuity, Lighthill derived a formula for calculating the acoustic power radiated by a unit volume of the turbulent region /568*

(1)

In order to use this formula, it is necessary to know all three components of the pulsation velocity of the flow, the correlation between these components, the volume of the characteristic eddy vortex and the frequency of the eddy pulsations. If these turbulence characteristics are known for the

*Numbers given in margin indicate pagination in original foreign text.

entire flow region, then the acoustic power of the jet may be calculated. However, since Lighthill states that most of the acoustic power is radiated by the initial section of the jet, it should be clear that a detailed study of turbulence characteristics is necessary in this particular region of the flow. Unfortunately, the available studies of this region (refs. 3 and 4) do not contain sufficient data for solving the Lighthill equations. For this reason, additional investigation of the turbulent characteristics of a free jet is required. The results of just such an investigation are given in the present paper, particular attention being given to the initial and transitional sections.

An air jet was studied flowing from nozzles with diameters of 40 and 55 mm at an initial rate of flow $u_0 = 25$-135 m/sec and temperature $T_0 = 288^{\circ}$K. Preliminary measurements using a Pitot tube showed that the mean velocity profiles in various sections of these jets coincide rather well with the universal profile (ref. 5).

The turbulence characteristics were measured with equipment made by the "Disa Electronics" company (ref. 6). This equipment consists of two constant-temperature hot-wire anemometers and a correlator. Two types of pickups were used for the measurements--a single-filament unit and an x-shaped unit. The sensing element of these pickups was a tungsten wire 0.005 mm in diameter and ~1 mm long. A spectrometer with a constant relative pass- /569 band of $^1/_{10}$ octave was used for spherical analysis of the turbulence. The equipment had a linear frequency response in the 16-20,000 cps range. Two single-filament pickups were used for measuring the longitudinal and transverse correlation coefficients of the longitudinal component of the pulsation velocity. One of these pickups was stationary, and the other was moved down-

stream along the axis of the jet (for measuring the longitudinal correlation coefficient) or along the normal to the axis toward the outer boundary of the jet (for measuring the transverse coefficient).

The integral scale of the turbulence was calculated from the formula

$$L = \int_0^{\xi_0} R\, d\xi, \quad R = \overline{u'(0)u'(\xi_0)} \Big/ \sqrt{\overline{u'^2}(0)}\sqrt{\overline{u'^2}(\xi_0)}. \qquad (2)$$

A study of the pulsation velocity fields showed that the curves for distribution of turbulence intensity in the initial and transitional sections of the jet $\sqrt{\overline{v'^2}}/u_0$; $\sqrt{\overline{u'^2}}/u_0$; $\sqrt{\overline{w'^2}}/u_0 = \varphi(y/r)$ have a maximum which remains constant in magnitude on the radial line passing through the edge of the nozzle (i. e. $y/r = 0$). A reduction is observed in the turbulence intensity maximum in the main section of the jet. For relatively small values of x (when $x/d < 20$), the law for reduction in maximum value of $\sqrt{\overline{u'^2}}/u_0$ along the mixing region of the main section of the jet may be expressed by $\sqrt{\overline{u'^2}}/u_0 = \text{const}\ d/x$.

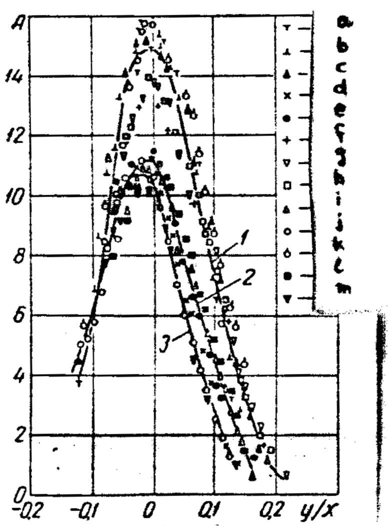

Fig. 1. Fields of the pulsation components of velocity in the initial and transitional sections of the jet (A in % $\equiv \sqrt{\overline{u'^2}}/u_0$--*1*; $\sqrt{\overline{w'^2}}/u_0$--*2*; $\sqrt{\overline{v'^2}}/u_0$--*3*): at $d = 40$ mm; $u_0 = 120$ m/sec (a--$x/d = 2$; b--4; c--5; d--6; e--4) and at

$d = 55$ mm; $u_0 = 100$ m/sec (f--$x/d = 3$; g--4; h--5; i--6; j--4; k--2; l--5; m--5)

It is interesting to note that the law for change in the maximum turbulence intensity along the mixing zone of the jet corresponds to the law for the change in the local mean velocity along the line passing through the edge of the nozzle. In the initial section of the jet, the local mean velocity on the radial line $y/r = 0$ remains constant (ref. 5), while in the main section of the jet (when $x/d < 20$) the local mean velocity varies according to the law $\frac{\text{const}}{\sqrt{x/d}}$. This means that the wake behind the output edge of the nozzle is an essential factor in formation of the turbulence field in the region of the jet under consideration.

It should also be pointed out that when the initial jet velocity is changed from 25 to 135 m/sec, there is almost no change in the turbulence intensity in the boundary layer of the jet.

Data from the measurements of all three components of the pulsation velocity given in dimensionless coordinates gave a satisfactorily accurate generalized curve describing each of these components (fig. 1). One of these curves for the main section is given in figure 2. It should be noted that although the curves are universal throughout the entire region of the boundary layer for the initial section of the jet, in the main section they are universal only in the region included between $y = 0$ and the outer boundary of the jet; it would be necessary to take the axis of the jet as the reference origin of the ordinates for full universality of the curves /570 in the main section of the jet. However, since it is more convenient for purposes of solving Lighthill's equations to give the turbulence characteristics throughout the entire jet region in a single coordinate system, co-

ordinates were selected in which the generalized curves are completely universal for the initial section (in view of the fact that most of the acoustic power is radiated by the initial section of the jet, according to reference 2).

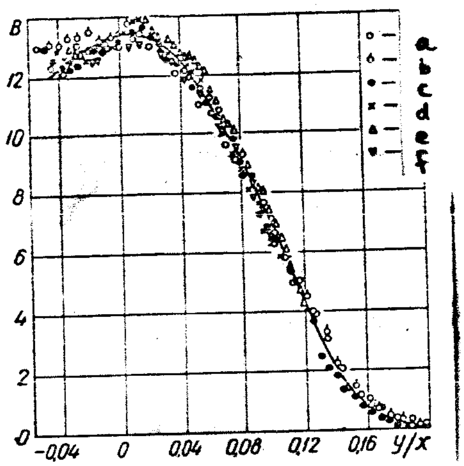

Fig. 2. Pulsation fields of the longitudinal component of velocity in the main section of the jet (B in % $\equiv$ $\equiv \sqrt{\overline{u'^2}}/u_0\sqrt{x/x_{\Pi}}$) at $d = 55$ mm; $u_0 = 100$ m/sec: a--$x/d = 6$; b--8; c--10; d--12; e--15; f--17

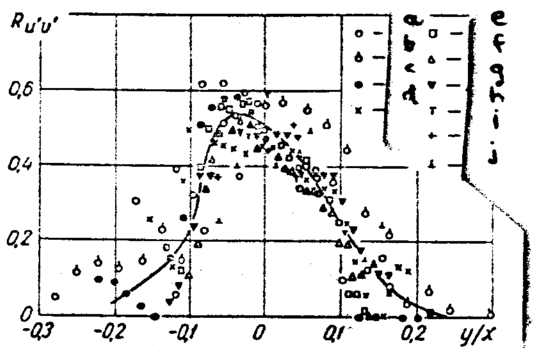

Fig. 3. Change in correlation coefficient $R_{u'v'}$ in the boundary layer of the jet at $d = 55$ mm; $u_0 = 100$ m/sec: a--$x/d = 1$; b--2; c--3; d--4; e--5; f--6; and at $d = 40$ mm; $u_0 = 120$ m/sec: g--$x/d = 4$; h--3; i--2; j--1

The coefficient $R_{u'v'}$ of correlation between the pulsations of the longitudinal and radial components of velocity measured in various sections of the

jet are given in figure 3. In spite of the considerable scatter in the points, the change in the correlation coefficient may also be described by a single universal curve. Measurement of the coefficient of correlation between the longitudinal and tangential components of the pulsation velocity showed that $R_{u'w'}$ in the boundary layer of the initial section is extremely small, and does not exceed 0.1 at a single point in this region.

It may be concluded from the results of these studies as well as from the data of other researchers that the integral scale of turbulence remains nearly constant across the mixing region of the jet, and increases linearly with axial distance x; the empirical relationships given in references 3 and 4 correspond rather well with the experimental data:

$$L_x = 0.13x, \ L_y = 0.036x.$$

Spectral analysis of the pulsation components of velocity showed that the nature of the turbulence spectrum tends to change somewhat with distance from the axis of the jet: there is an increase in the fraction of low frequency components and a reduction in the fraction of high frequency components. However, the form of the spectral curves is changed only slightly, and it may be assumed that the spectrum in the transverse section of the mixing region of the jet remains constant. As the distance increases between the section being studied and the nozzle cutoff, there is a considerable change in the nature of the turbulence spectrum (fig. 4). A change in the initial flow velocity has an effect which is just as great. It is also obvious from the graphs given in figure 4 that the spectra of all three components of the pulsation velocity are rather close in nature at the same flow point.

Analysis of the data showed that the turbulence spectra may be /571 given in dimensionless form if the Strouhal number Sh is used as the dimension-

less frequency. For the initial and transitional sections of the jet, Sh = $= f_x/u_0 = 1.35$, and for the main part of the jet Sh $= fx\sqrt{x/x_n}/u_0 = 1.35$.

If the width of the boundary layer in the given section of the jet is taken as the characteristic geometric dimension instead of the axial distance x, we get Sh≈0.365.

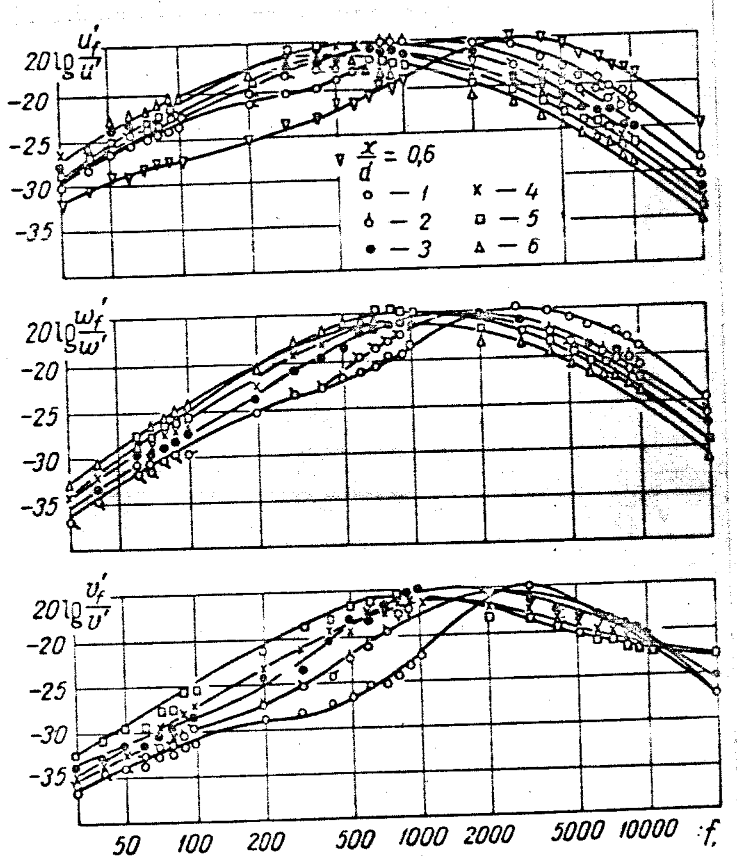

Fig. 4. Spectra of the longitudinal, tangential and radial components of velocity in the initial and transitional sections of the jet at $d = 55$ mm; $u_0 = 100$ m/sec

The acoustic power radiated by the initial section of the jet is

$$\Delta W \sim V f^4 T_{ij}^2 / 4\pi a_0^5 \rho_0. \tag{3}$$

The initial section of the jet consists of a nucleus of constant velocities and a boundary layer. We shall assume, as did Lighthill, that no noise

is created by the nucleus of constant velocities. This also follows from the results of these studies which indicate that the pulsation velocities in this region are insignificant, and consequently the noise intensity determined from equation (1) is also small. Therefore, the ordinate of the inner boundary of the jet y_2 may be taken as the lower limit of integration in formula (3). /572

Since the size of a typical vortex is determined by the region of positive correlation, we may write

$$V_l = L_x L_y L_z = k_1 x^3. \tag{4}$$

If we use formulas (1) and (4) and the value of Sh, then equation (3) for the initial section of the jet may be written as

$$W_x = k \frac{u_0^4 dx}{\rho_0 a_0^5 x} \int_{y_2}^{y_1} T_{ij}^2 (r + y)\, dy. \tag{5}$$

Since all three components of the pulsation velocity and the coefficients of correlation between them can be given in the form of dimensionless universal curves, the integral in equation (5) may be easily calculated graphically. Then equation (5) takes the form

$$W_x = k \frac{\rho_0 u_0^8 dx}{a_0^5} (a_1 r + a_2 x). \tag{6}$$

Consequently, the power of the noise radiated by the initial section of the jet is defined by the expression

$$W_1 = k \frac{\rho_0 u_0^8}{a_0^5} \int_0^{x_H} (a_1 r + a_2 x)\, dx = k \frac{\rho_0 u_0^8 d^2}{a_0^5} (2a_1 + 8a_2), \tag{7}$$

where $x_H = 4d$.

In a similar manner, we may calculate the acoustic power radiated both by the elementary section and by the entire region of the main and transitional sections of the jet.

For the main section of the jet

$$W_x = k\frac{\rho_0 u_0^8 x_\text{п}^4 dx}{a_0 x^5}(a_3 r^2 + a_4 r x + a_5 x^2), \tag{8}$$

$$W_2 = k\frac{\rho_0 u_0^8 d^2}{a_0^5}\left(\frac{a_3}{16} + a_4 + 18 a_5\right), \tag{9}$$

if $x_\text{п}$ is taken as $6d$.

For the transitional section of the jet

$$W_x = k\frac{\rho_0 u_0^8 dx}{a_0^5}\left(\frac{a_3 r^2}{x} + a_4 r + a_5 x\right), \tag{10}$$

$$W_3 = k\frac{\rho_0 u_0^8 d^2}{a_0^5}(0.101 a_3 + a_4 + 10 a_5). \tag{11}$$

The coefficients in formulas (6)-(11) determined by graphic integration are

$$a_1 = 0.148\cdot 10^{-3};\quad a_2 = 0.525\cdot 10^{-5};\quad a_3 = 0.398\cdot 10^{-3};$$
$$a_4 = 0.561\cdot 10^{-4};\quad a_5 = 0.192\cdot 10^{-5}.$$

The acoustic power radiated by the entire jet,

$$W_x = 2\pi\, dx \int_{y=-r}^{y_1} (r + y)\, dy\, \Delta W. \tag{12}$$

Calculation by formula (12) shows that the acoustic power radiated by /573 the initial and transitional sections of the jet makes up approximately 75% of the total acoustic power of the jet. Thus most of the noise is radiated by sections of the jet located less than 6 jet diameters from the nozzle cutoff.

Formulas (6), (8) and (10) may be used for calculating the acoustic power spectrum of the jet. Since the radiated acoustic power is proportional to the fourth power of the pulsation velocity, the spectrum for the acoustic power radiated by a given section of the jet will have a maximum which is more sharply defined than the maximum in the turbulence spectrum. Therefore it may be assumed that the spectrum for the acoustic power radiated by the elementary section comprises a rather narrow band with an average frequency which corresponds to that of the maximum component of the turbulence spectrum. The relationship between this average frequency and the distance x is expressed by the values of the number Sh.

This type of calculation for the acoustic power spectrum of a jet 100 mm in diameter showed satisfactory agreement with the data of experimental studies (ref. 7).

Notation

W--acoustic power; ρ_0--atmospheric density of the air; V_l--volume of a typical vortex; f--pulsation frequency; T_{ij}--tensor of turbulent friction stress; $d = 2r$--nozzle diameter; u_0--initial flow velocity; T_0--total flow temperature; u', v', w'--longitudinal, radial and tangential components respectively of the pulsation velocity; L--integral scale of turbulence; R--coefficient of correlation between the longitudinal components of pulsation flow velocity u' measured at two points separated by distance ξ; ξ_0--distance between the filaments in the hot-wire anemometer pickup where R vanishes; y--distance in the transverse section of the jet from the edge of the nozzle to the point being studied, the positive direction being taken as that toward the outer boundary of the jet; x--axial distance from the edge of the nozzle to the point being studied; $R_{u'v'}$, $R_{u'w'}$--relative coefficients of correlation between the corresponding two components of the pulsation velocity; L_x, L_y--longitudinal and transverse scales of turbulence respectively; x_{H}, x_{Π}--abscissas of the ends of the initial and transitional sections of the jet respectively; y_1, y_2--ordinates of the outer and inner boundaries of the jet.

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